Spatial distribution of GRB with known redshifts

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The problem statement

the theme works

Ming-Hua Li and Hai-Nan Lin

A&A 582, A111 (2015)
`https://arxiv.org/abs/1509.03027v1`
244 SWIFT GRB with $r_0 = 387.51 \pm 132.75$ h^{-1} Mpc,
$\gamma = 1.57 \pm 0.65$ (at 1σ confidence level)
homogeneous scale $r \geq 7700$ h^{-1} Mpc.

A. A. Raikov, V. V. Orlov, and O. B. Beketov are proposed the pairwise distances method for estimate of fractal dimension

Astrophysics, Volume 53, Issue 3, pp.396-408 (2010)
`https://arxiv.org/abs/1001.4592`
201 long GRB with $D = 2.2 \div 2.5$ on small scales.
GRB is indicators of large scale structures. Group of five GRB on $23h\,50m < \alpha < 0h\,50m, 5^\circ < \beta < 25^\circ$ & $0.81 < z < 0.97$.

The problem statement
The purposes

The main scientific problems

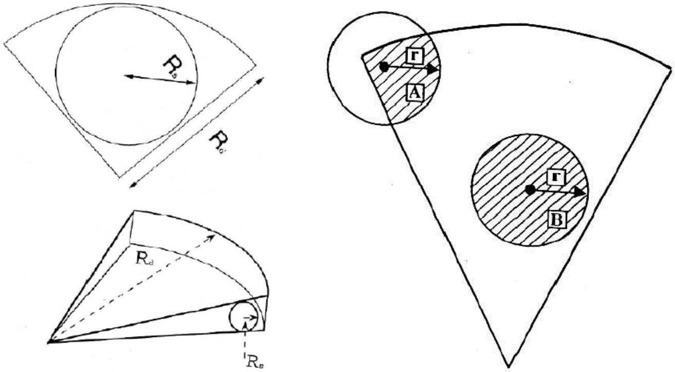
- the construction of distributions of the GRB catalog;
- the estimating of fractal dimension by the conditional density method;
- the estimating of fractal dimension by the pairwise distances method.

The main technical problems

- the creating of software for correct statistical analysis of any samples;
- the verification of fractal dimension of Cantor's sets and uniform sets; taking into account the luminosity function and limited geometry.

Integral, conditional, density (concentration)

Sylos Labini et al., Phys.Rep., 293, 66, 1998



Methods
the integral conditional density method

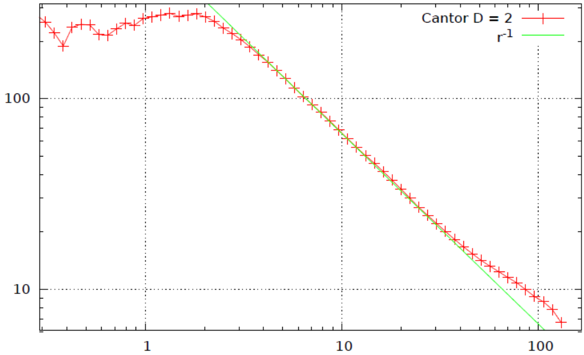


Figure: the conditional density distribution for Cantor's set D=2.0.

Methods

the integral conditional density method

$$\Gamma^*(r) = \langle n(r' < r) \rangle_p$$

$$n(r) = \frac{1}{N_c(r)} \sum_{i=1}^{N_c(r)} \frac{N_i(r)}{V(r)},$$

- $\langle ... \rangle_p$ is a averaging over *conditional* sample points
- $N_c(r)$ – a number of balls within a set geometry
- $N_i(r)$ – a number of points within ball
- $V(r)$ – a volume of ball

The distance of pairwise is length of the segment between two set points in accordance with set geometry.

$$f(l) = Dl^{D-1}(L/2)^{-D} I_\mu(\frac{D+1}{2}, \frac{1}{2}),$$

from Kendall & Moran (1963)

- D – an integer space dimension
- l – a segment length
- L – a maximal length of segments
- $I_\mu(p, q)$ – incomplete Bessel's function
- $\mu = 1 - l^2/L^2$

Methods
the pairwise distances method

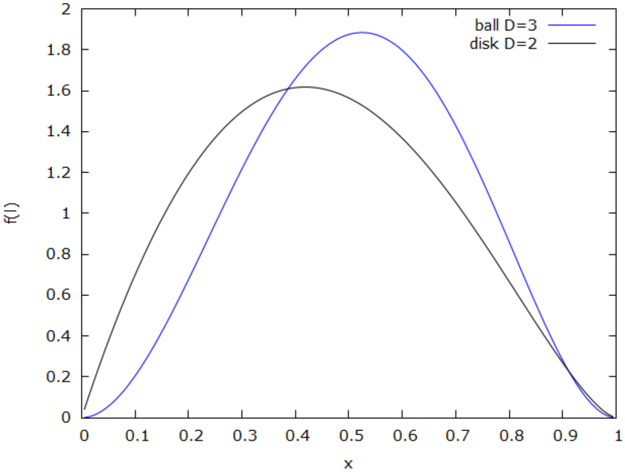


Figure: the analytical distributions for D=3.0 & D=2.0.

For D=3

$$f(l) = \frac{12}{L^6} l^2 (L - l)^2 (2L + l)$$

with an asymptote at $l << L$

$$f(l) \sim l^{D-1}$$

This asymptote is stabled for not only integer dimension, but also for fractional dimension with any geomentry of set.
A. A. Raikov and V. V. Orlov, MNRAS, 418, 2558–2564 (2011)
https://academic.oup.com/mnras/article-lookup/doi/10.1111/j.1365-2966.2011.19645.x

The source GRB sample

included 364 objects

the main source of redshifts and fluence

The Swift Gamma-Ray Burst Mission
https://swift.gsfc.nasa.gov/archive/grb_table/

have been supplemented by

- Fa-Yin Wang, Shi Qi & Zi-Gao Dai
  MNRAS, Volume 415, Issue 4, pp. 3423-3433 (2011)
  https://arxiv.org/pdf/1105.0046v2.pdf
- J. S. Wang, F. Y. Wang, K. S. Cheng, & Z. G. Dai
  A&A 585, A68 (2016)
  https://arxiv.org/pdf/1509.08558v2.pdf
- founded and maintained by Tilan Ukwatta
  http://www.grbcatalog.org

The source GRB catalog
included 364 objects

name	l	b	T_{90}	F_{obs}	z	program
151215A	177.25358	8.55309	17.80	3.10	2.590	SWIFT
150423A	9.70821	59.24722	0.22	0.63	1.394	SWIFT
141121A	200.39117	26.85321	549.90	53.00	1.470	SWIFT
...						

Table: the initial catalog columns

X_{Mpc}	Y_{Mpc}	Z_{Mpc}	R_{Mpc}	lgS_{obs}	lgL_b
-5862.4	281.2	882.7	5935.1	-0.75	51.97
2099.7	359.2	3580.2	4166.0	0.45	52.53
-3605.5	-1340.3	1947.5	4311.5	-1.01	51.11
...					

Table: the processed catalog columns by the FDE program

The source GRB catalog

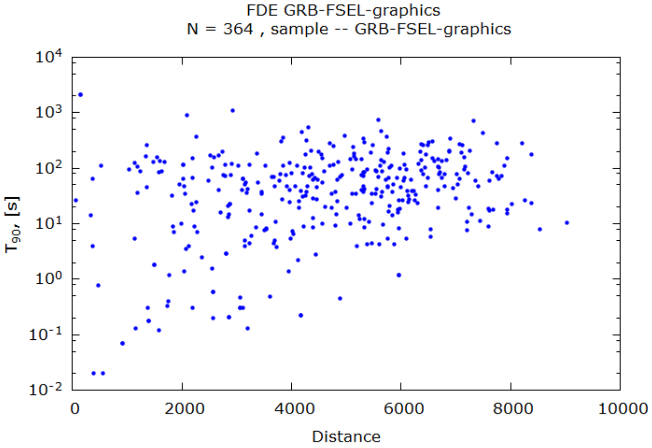


Figure: The distribution of T_{90} vs z for all GRBs is correlated with D. Kocevski & V. Petrosian, AJ, 765, 116 (2013).

Formulas

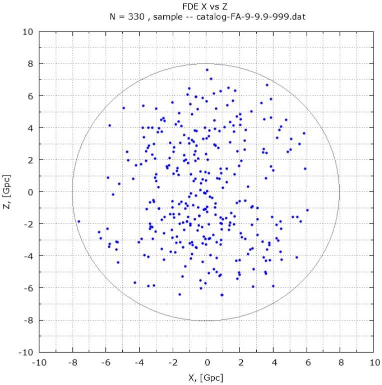
metric distance for the non-interacting two-fluid dust-vacuum Friedmann model to object

$R(z)_{Mpc} = \frac{c}{H_0} \int_0^z (\Omega_v^0 + \Omega_m^0 (1+z')^3 - \Omega_k^0 (1+z')^2)^{-1/2} dz'$,
where $H_0 = 70$ km s^{-1}Mpc^{-1} is Hubble's constant,
$c = 3 \cdot 10^{10}$ sm s^{-1}, $\Omega_v^0 = 0.7$, $\Omega_m^0 = 0.3$, $\Omega_k^0 = 0$, z is redshift.

luminosity

$L(z) = 4\pi S_{obs} R(z)^2_{sm} (1+z)^n$, where S_{obs} [erg s^{-1} sm^{-2}] is flux that equals to the ratio of a fluence from 15 to 150 keV to a T90-time, and z is redshift; n corresponds different cosmological models, we take $n = 2$ (ΛCDM)

Two GRB subsamples
included 325 & 297 objects respectively (from 364)



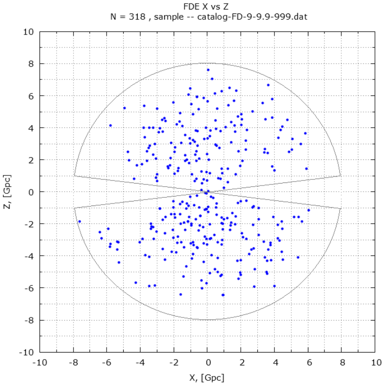


Figure: the GRB metric coordinates projections to X-Z plane.

The Algorithm

The Fractal Dimension Estimater software (FDE)

the model uniform or fractal catalogue (MUC & MFC)

- generation of a uniform or fractal set within the cube [-1:1] and luminosity assignment to every point according to luminosity function;
- limitation by geometry and maximal distance;
- selection by visible magnitude for Malmquist bias modeling;

the determination of fractal dimension

- bar charts construction of density (CD & MD);
- reduction of the GRB sample & MFC by MUC;
- detection of slope that is the approximated fractal dimension (AFD)

uniform subsamples

included 324 & 323 objects respectively

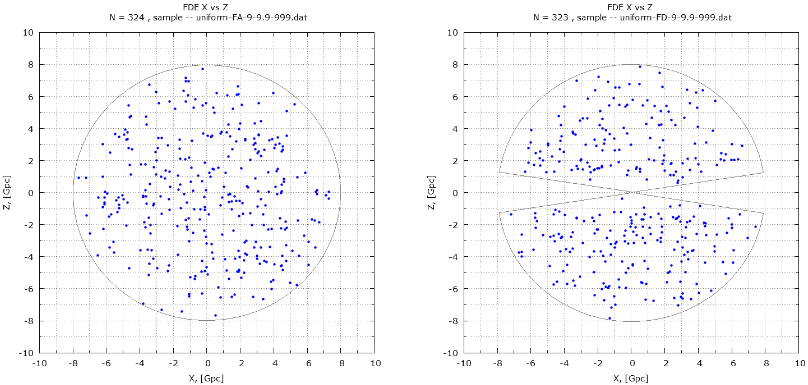


Figure: the uniform set metric coordinates projections to X-Z plane.

Model catalogs

comparison of the GRB sample with model uniform distribution

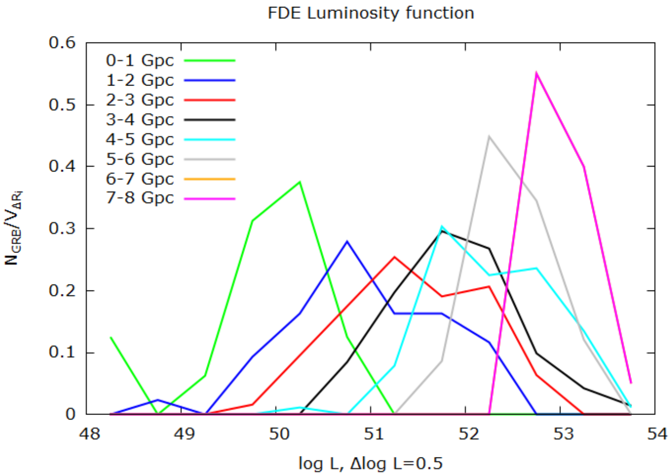


Figure: The GRB luminosity functions for ΔR.

Model catalogs

comparison of the GRB sample with model uniform distribution

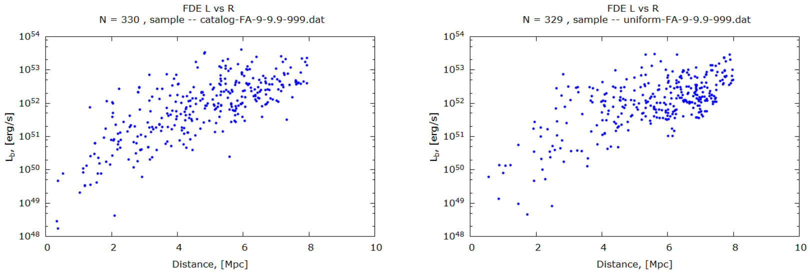


Figure: The GRB & the model uniform distributions of L vs R.

The results

Reduced conditional density method, CD

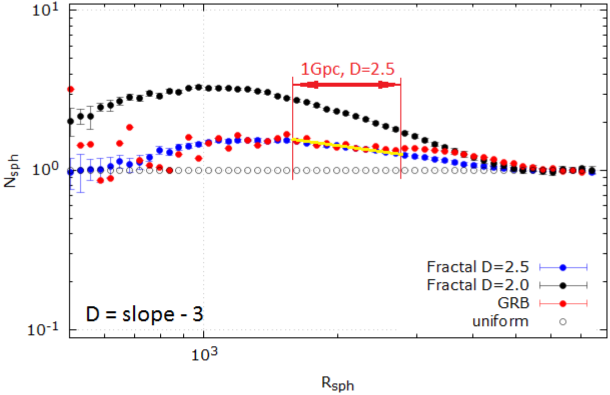


Figure: The GRB distribution (red) comparing to model fractal catalogs for $D = 2.0$ (black) & $D = 2.5$ (blue) in case of entire celestial sphere.

The results

Reduced pairwise distances method, MD

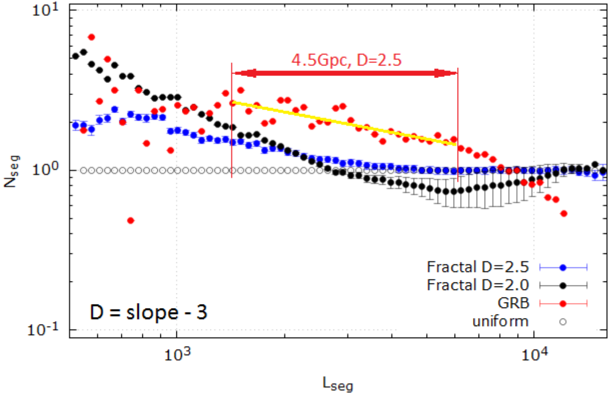


Figure: The GRB distribution (red) comparing to model fractal catalogs for $D = 2.0$ (black) & $D = 2.5$ (blue) in case of entire celestial sphere.

- The Fractal Dimension Estimater is created. The program is a powerful tool of statistical analysis of any sets with known geometry.
- The model fractal and uniform catalogs are generated by the FDE program. An error of approximated fractal dimension (AFD) considering the luminosity selection is less than 0.1 or 5% of value.
- The GRB AFD is 2.6 ± 0.15 on $R = 1.5 \div 2.5$ Gpc by the CD method.
- The GRB AFD is 2.6 ± 0.10 on $R = 1.5 \div 6.0$ Gpc by the MD method.
- The GRB AFD for the sample without Galactic belt is 2.6 ± 0.10 on $R = 1.5 \div 4.0$ Gpc by the MD method

The galactic map

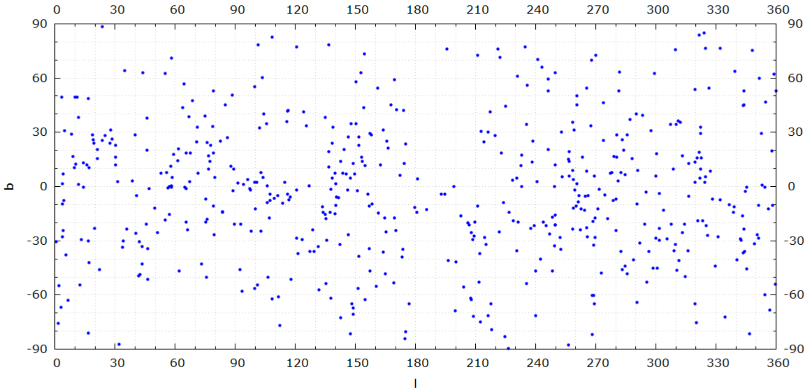


Figure: A model fractal catalogue for $D = 2.5$ and a degree of hierarchy $H_d = 5$.

Thank you for your attention