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# Neutrino interaction with nuclei

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**Abstract** Cross section of neutrino-nucleus interaction is calculated for a number of nuclei, used in neutrino detectors. The calculations are performed by the means of the model-independent method, based on the experimental data on nuclear reactions. The obtained theoretical values coincide with the existing experimental results.

**Keywords:** Neutrino Detection, Charge Current and Neutral Current Interaction, Charge Exchange Reactions, Nuclear Resonance Fluorescence

## 1. Introduction

Theoretical investigation of neutrino-nucleus interaction has many applications, such as neutrino detection, neutrino oscillations study, nucleosynthesis processes examination. The expressions for cross sections contain nuclear matrix elements, which are the goal of calculations for a variety of nuclear models. For there is a certain spread of results of these estimations it is reasonable to obtain nuclear matrix elements by the model-dependent approach, which use experimental data on nuclear reactions. These are beta decay processes, charge exchange reactions, nuclear resonance fluorescence, which can give direct information on nuclear structure. Below the corresponding model-independent cross section calculations are produced and compared with existing experimental data on neutrino-nucleus interaction.

## 2. Charged Channel

In neutrino interaction with nucleus, caused by charged current, the following transition takes place:

$$\nu_l + (A, Z) \rightarrow (A, Z + 1) + l^- \quad (1)$$

The final nucleus  $(A, Z + 1)$  can be in ground or excited state. There are experimental data for cross section of reaction (1) for two nuclei:  $^{12}\text{C}$  with  $\nu_e$  and  $\nu_\mu$  beams and  $^{56}\text{Fe}$  with  $\nu_e$  beam, obtained by KARMEN and LSND Collaborations. The exclusive reaction  $k - j \rightarrow k - j$

$$\nu_e + ^{12}\text{C} \rightarrow ^{12}\text{N}_{g.s.} + e^- \quad (2)$$

was investigated both at KARMEN [1,2] and LSND [3]. The neutrino source is the positive muon decay at rest (DAR),  $\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu$ . The corresponding spectra of  $\nu_e$  and  $\bar{\nu}_\mu$ , the Michel spectra, have the form.

$$S_M(E_{\nu_e}) = \frac{96E_{\nu_e}^2}{m_\mu^4} (m_\mu - 2E_{\nu_e}), \quad (3)$$

$$S_M(E_{\bar{\nu}_\mu}) = \frac{32E_{\bar{\nu}_\mu}^2}{m_\mu^4} \left(\frac{3}{2}m_\mu - 2E_{\bar{\nu}_\mu}\right)$$

The maximal neutrino energy is  $m_\mu/2=52.8$  MeV. The monoenergetic muon neutrino flux with  $E_{\nu_\mu} = 29.8$  MeV, originating from stopped positive pion decay,  $\pi^+ \rightarrow \mu^+ + \nu_\mu$ , is also present.

The values of cross section of  $^{12}\text{C}(\nu_e, e^-)^{12}\text{N}_{g.s.}$ , averaged over electron neutrino spectrum (3) are the following:

$$\langle \sigma \rangle = (8.1 \pm 0.9_{stat} \pm 0.75_{syst}) \cdot 10^{-42} \text{ cm}^2 [1],$$

$$\langle \sigma \rangle = (9.1 \pm 0.5_{stat} \pm 0.8_{syst}) \cdot 10^{-42} \text{ cm}^2 [2],$$

$$\langle \sigma \rangle = (9.1 \pm 0.4_{stat} \pm 0.9_{syst}) \cdot 10^{-42} \text{ cm}^2 [3].$$

Theoretical calculations of type (2) reaction cross section can be performed on the base of experimental data on  $\log ft$  value of  $\beta$ -transition from the final to the initial nucleus [4]. The corresponding expression is:

$$\sigma(\varepsilon_\nu) = \frac{2 \ln 2 (2J_f + 1) \pi^2}{m_e^3 \cdot 10^{\log(ft)_{\beta^+, EC}}} \pi_r \varepsilon_r F(Z_f, \varepsilon_r) \quad (4)$$

$$= \frac{0.264 (2J_f + 1)}{10^{\log(ft)_{\beta^+, EC}}} \pi_r \varepsilon_r F(Z_f, \varepsilon_r) \cdot 10^{-40} \text{ cm}^2$$

Here  $J_f$  is the total momentum of the final nucleus,  $J_f=1$  for  $^{12}\text{N}_{g.s.}$ ;  $\log(ft)_{\beta^+, EC}$  is related to the  $\beta$  transition from the final to initial nucleus;  $\varepsilon_\nu$ ,  $\pi_r$ ,  $\varepsilon_r$  are the neutrino energy, momentum and energy of outgoing electron in units of electron mass  $m_e$  respectively;  $\varepsilon_r = \varepsilon_\nu - M_f + M_i$ , where  $M_i$  and  $M_f$  are the masses of the initial and final nuclei in units of electron mass; and  $F(Z_f, \varepsilon_r)$  is the Coulomb correction function. The method of calculation of  $F(Z_f, \varepsilon_r)$  is presented in [5]. The neutrino threshold energy is  $\varepsilon_{\nu,thr} = M_i - M_f + 1$ . For  $^{12}\text{C}(\nu_e, e^-)^{12}\text{N}_{g.s.}$  reaction  $E_{\nu,thr}=17.3$  MeV. For  $^{12}\text{N}(\beta^+)^{12}\text{C}$  transition  $\log ft_{\beta^+} = 4.12 \pm 0.03$  [6]. As a result the cross section (4) averaged over  $\nu_e$  spectrum (3) equals  $9.1 \cdot 10^{-42} \text{ cm}^2$ , and coincides with experiment.

The  $^{12}\text{C}(\nu_\mu, \mu^-)^{12}\text{N}_{g.s.}$  reaction cross section was measured in LSND experiment. The  $\nu_\mu$  beam is produced by the  $\pi^+$  decay in flight (DIF). The muon neutrino spectrum has its maximum at  $E_\nu \sim 70$  MeV and extends to  $\sim 300$  MeV. The flux-averaged cross section is  $(6.6 \pm 1.0_{stat} \pm 1.0_{syst}) \cdot 10^{-41} \text{ cm}^2$ . Theoretical expression for cross section of  $\nu_\mu + ^{12}\text{C} \rightarrow ^{12}\text{N}_{g.s.} + \mu^-$  is determined by the formula, similar to (4)

$$\sigma(\varepsilon_\nu) = \left(\frac{m_\mu}{m_e}\right)^2 \frac{2 \ln 2 (2J_f + 1) \pi^2}{m_e^3 \cdot 10^{\log ft_{\beta^+}}} \pi_r \varepsilon_r F(Z_f, \varepsilon_r) \quad (5)$$

Here neutrino energy, momentum and energy of outgoing electron are scaled by muon

mass  $m_\mu$ . The muon neutrino threshold energy equals  $E_{\nu,thr}=122.4$  MeV. Theoretical value of cross section (5), averaged over muon neutrino DIF spectrum,  $\langle\sigma(\varepsilon_\nu)\rangle=8.6\cdot 10^{-41}$  cm<sup>2</sup> and is in agreement with experimental result.

For there is reasonable accord between theoretical estimations and experimental measurements of neutrino-<sup>12</sup>C interaction in charged channel, the expression (4) can be used for analysis of neutrino signal, registered with the help of liquid scintillator, which contain large quantity of carbon-12 nuclei. Particularly, for  $^{12}C(\nu_e, e^-)^{12}N_{g.s.}$ ,  $E_{\nu_e} = 40$  MeV,  $\sigma(E_{\nu_e}) = 14.4\cdot 10^{-42}$  cm<sup>2</sup>.

Investigation of neutrino-<sup>56</sup>Fe interaction is an actual problem, due to the presence of large amounts of iron as a shielding material in scintillator detectors, such as LSD and LVD. Electrons and gamma quanta would be produced under the exposure of supernova neutrino radiation in the reaction with iron nuclei and they could be recorded by the detector [7]. Charge current reaction is the following:



In reaction (6) cobalt nucleus is in the excited state, for the ground state of cobalt-56 nucleus has quantum numbers  $4^+$ , so the corresponding cross section for  ${}^{56}Fe(\nu_e, e^-){}^{56}Co_{g.s.}$  is small, compared to the cross section of giant resonances excitation. These resonances are: analog  $0^+$  resonance (AR), caused by Fermi transition, and Gamow-Teller  $1^+$  (GT) resonances. The cross sections (6) can be calculated by the means of the expressions:

$$\sigma_F(E_\nu) = \frac{G_\beta^2 m_e^2}{\pi} M_F^2 \pi_e \varepsilon_e F(Z_f, \varepsilon_e)$$

$$\sigma_{GT}(E_\nu) = \frac{G_\beta^2 m_e^2}{\pi} g_A^2 M_{GT}^2 \pi_e \varepsilon_e F(Z_f, \varepsilon_e)$$

Here  $M_F$  and  $M_{GT}$  are nuclear matrix elements,  $g_A$  is the axial-vector interaction constant,  $g_A=1.2761$ ,  $\varepsilon_e$  and  $\pi_e$  are the energy and momentum of outgoing electron in units of  $m_e$ ,  $\varepsilon_e=(E_\nu-\Delta)/m_e$ ,  $\Delta$  is the mass difference of the  ${}^{56}Co^*$  and  ${}^{56}Fe$  nuclei. For AR nuclear matrix element can be written as follows:  $M_F^2=(N-Z)$ . Gamow-Teller matrix elements obey sum rule, which in application to neutron-rich nuclei can be expressed as  $\sum_i M_{GT_i}^2 = 3(N-Z)e_q^2$ , where  $e_q$  is the effective GT charge,  $e_q=0.8$ . Thus nuclei with large

neutron excess (N-Z) are preferable for neutrino detection. In [8] a number of  $1^+$ -states and the appropriate matrix elements were found for the reaction (6) on the base of theory of Gamow-Teller resonance [9]. Excitation of AR and GT-resonances in  ${}^{56}Co$  produces 5-10 MeV gamma quanta accompanied by electron emission. For the test of the nuclear model the total  ${}^{56}Fe(\nu_e, e^-){}^{56}Co^*$  reaction cross section, averaged over muon DAR neutrino spectrum was calculated. It gives the value  $2.62\cdot 10^{-40}$  cm<sup>2</sup>, which is in agreement with results, obtained in KARMEN experiment,  $(2.56 \pm 1.08(stat) \pm 0.43(syst))\cdot 10^{-40}$  cm<sup>2</sup> [10].

The obtained cross section is used for estimation of number of neutrino signals from SN1987a, observed in LSD [11], which correspond to the first stage of rotating mechanism of Supernova explosion scenario [12]. The neutrino flux during the first burst consists of electron neutrinos with a total energy  $W_\nu = 8.9 \times 10^{52}$  erg. The neutrino energy spectrum is hard with an average energy of ~30–40 MeV. The second neutrino burst [13] corresponds to

the standard collapse theory without rotation with the formation of the neutrino sphere and with an equal energy distribution between all types of neutrinos [14]. The calculated event number [8,15] coincides with the observed number of signals, registered in LSD.

Distribution of Gamow-Teller strength  $B(GT)$  for transition (6),  $B(GT) = M_{GT}^2$ , can be deduced from the experiments on charge exchange  $^{56}\text{Fe}(p,n)^{56}\text{Co}$  reaction [16]. Several  $1^+$  states below excitation energy  $E_x \sim 4$  MeV were observed and a broad maximum with great density of GT-states in the  $E_x$  region 8-15 MeV was found. The total GT-strength is  $\Sigma B(GT) = 9.9 \pm 2.4$  [16]. This leads to reaction (6) averaged cross section  $\langle \sigma \rangle = (3.08 \pm 0.5) \cdot 10^{-40} \text{ cm}^2$ , which agrees with KARMEN experiment. For  $E_\nu = 40$  MeV cross section of  $^{56}\text{Fe}(\nu_e, e^-)^{56}\text{Co}$ , calculated on the base of charge-exchange reaction results [16], equals  $4.8 \cdot 10^{-40} \text{ cm}^2$ .

Charge-exchange reaction on  $^{71}\text{Ga}$  [1] can be used for calculation of solar-neutrino absorption cross section for the Gallium-Germanium experiment [18].

### 3. Neutral Channel

Calculation of inelastic neutrino scattering cross section on nuclei, caused by neutral current is valuable to investigation of neutrino oscillations, processes during Supernova explosions and to neutrino detectors construction. These reactions are equally sensitive to all neutrino flavors:

$$\nu_x + (A, Z) \rightarrow \nu'_x + (A, Z)^* \quad (6)$$

where  $x=e, \mu, \tau$ . In the energy range under consideration the inelastic scattering is determined, in the main, by allowed transitions. For the ground state of the initial nucleus the cross section of (6) is expressed as follows:

$$\sigma^{NC}(E_\nu) = \frac{G_F^2 g_A^2}{\pi(2J_i + 1)} (E_\nu - E_x)^2 \left\langle f \left\| \sum_k \boldsymbol{\sigma}(k) t_0(k) \right\| i \right\rangle^2 \quad (7)$$

Here  $E_\nu$  is the incident neutrino energy,  $E_x$  is the excitation energy of nucleus  $(A, Z)^*$ . The Gamow-Teller strength  $B(GT_0)$  handles the dependence of cross section on nuclear structure.

$$B(GT_0) = \frac{g_A^2 \left\langle f \left\| \sum_k \boldsymbol{\sigma}(k) t_0(k) \right\| f \right\rangle^2}{2J_i + 1} \quad (8)$$

In (8)  $t_0$  is the zero component of isospin operator and summation is performed over all nucleons of the nucleus.

The magnitude of  $B(GT_0)$  can be obtained by the model independent way on the base of experimental data. For allowed GT-transitions  $\Delta J=1$ ,  $\Delta \pi=0$ , so electromagnetic  $B(M1)$  strengths can give information on nuclear matrix elements, which govern neutral current neutrino-nucleus interaction [20]. Electromagnetic dipole transitions in  $^{56}\text{Fe}$  [21] and in  $^{208}\text{Pb}$  [22] were measured in photon-scattering experiments with linearly polarized photon beam, which give the possibility to determine the parity quantum numbers of excited dipole states and corresponding magnetic dipole M1 strength. These results were used in [15, 23] for calculation of cross section of inelastic neutrino scattering on  $^{56}\text{Fe}$  and  $^{208}\text{Pb}$ . In the case of

$^{208}\text{Pb}$  the experiment on investigation of resonance structure of  $^{207}\text{Pb}+n$  system [24] was also taken into account.

For M1 transitions the width of the excited  $1^+$ -state relative to the transition of the nucleus to the ground state is determined by the expression [25]

$$\Gamma_0 = \frac{16\pi}{27} \frac{E_x^3}{\hbar^3 c^3} B(M1) \quad , \quad (9)$$

where  $E_x$  is the excitation energy and  $B(M1)$  is the reduced probability. It can be shown, that isovector contribution dominates in  $B(M1)$  [20] and  $B(GT_0)$  and  $B(M1)$  are connected by the following relation:

$$B(GT_0) = \frac{4\pi g_A^2}{3\mu_v^2} \frac{B(M1)}{\mu_N^2} \quad (10)$$

Here  $\mu_v$  is the isovector nucleon magnetic moment,  $\mu_v=4.706$ .

It follows from (9), (10), that

$$\frac{B(M1)}{\mu_N^2} = 0.2592 \frac{\Gamma_0}{E_x^3} \quad ,$$

where excited state width  $\Gamma_0$  is measured in meV and  $E_x$  in MeV and

$$B(GT_0) = 0.308 \frac{B(M1)}{\mu_N^2} \quad .$$

Thus  $B(GT_0)$  can be obtained from the values of excitation energy and width of  $1^+$ -state and cross section of inelastic neutrino scattering can be obtained according to the following expression:

$$\sigma^{NC}(E_\nu) = 1.6862 \cdot 10^{-44} (E_\nu - E_x)^2 B(GT_0) \quad (11)$$

Here  $E_\nu$ ,  $E_x$  are measured in MeV.

Cross section of inelastic neutrino-nucleus interaction, governed by a neutral current was measured for excitation of ( $1^+$ , 1; 15.11 MeV) state in  $^{12}\text{C}$  nucleus in the KARMEN experiment. The results of flux averaged cross sections for electron neutrino and muon antineutrino spectrum (3) are:

$$\langle \sigma^{NC}(\nu_e + \bar{\nu}_\mu) \rangle = (10.8 \pm 5.1_{\text{stat}} \pm 1.1_{\text{syst}}) \cdot 10^{-42} \text{ cm}^2 \quad [26];$$

$$\langle \sigma^{NC}(\nu_e + \bar{\nu}_\mu) \rangle = (10.4 \pm 1.0_{\text{stat}} \pm 0.9_{\text{syst}}) \cdot 10^{-42} \text{ cm}^2 \quad [2].$$

The experimental value of ( $1^+$ , 1; 15.11 MeV) state width equals,  $\Gamma_0=38.5 \pm 0.8$  eV [27]. So, from (9)-(11) the value of  $\sigma^{NC}$  can be calculated,  $\langle \sigma_{th}^{NC}(\nu_e + \bar{\nu}_\mu) \rangle = 14.7 \cdot 10^{-42} \text{ cm}^2$ . The measured cross section for monoenergetic  $\nu_\mu$  from  $\pi^+$ -decay at rest,  $E_{\nu_\mu}=29.8$  MeV, is:  $\sigma^{NC} = (3.2 \pm 0.5_{\text{stat}} \pm 0.4_{\text{syst}}) \cdot 10^{-42} \text{ cm}^2$  [28]. Theoretical value of this cross section, based on intensity of M1  $\gamma$ -transition [27], according to (9)-(11), equals  $3.2 \cdot 10^{-42} \text{ cm}^2$ . Consequently, the addressed approach leads to satisfactory agreement with experimental data for inelastic neutrino scattering on  $^{12}\text{C}$ . The calculated magnitude of  $^{12}\text{C}(\nu, \nu')^{12}\text{C}^*(1^+, 1; 15.1 \text{ MeV})$  cross section for  $E_\nu=40$  MeV is  $9.3 \cdot 10^{-42} \text{ cm}^2$ .

## 4. Conclusion

The model-independent approach, based on nuclear reactions investigation, gives the possibility to determine nuclear matrix elements and estimate cross sections of neutrino-nucleus interaction. The calculated values coincide with the results of KARMEN and LSND experiments for and  $^{12}\text{Fe}$   $^{12}\text{C}$  nuclei. The extension of this experimental work for a more wide set of nuclei, both in neutral and charge channels is a valuable problem for elementary particle physics. High-precision experiments on charge-exchange reactions for a number of nuclei of interest are desirable for exact determination of nuclear excitation characteristics. The method under consideration can be used for calculation of neutrino-nuclei interaction cross section in application to different problems of neutrino physics.

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