Bull. Spec. Astrophys. Obs., 1996, 39, 87–100

Modelling of integral spectra of galaxies by the method of population synthesis

O. A. Zenina

Rostov State University

Received July 6, 1995; accepted September 8, 1995.

Abstract.
A detailed description of a population synthesis method for modelling the content of stellar populations in complex systems, galaxies and star clusters, from their integral spectra is given. As the original database for modelling, two banks of reference observational stellar spectra, of Pickles and Jacoby, have been selected. The method allows one to define the following physical characteristics of a stellar system: detailed composition of a stellar population, average metallicity of the system and metallicities of populations it comprises, age of the last star formation in the system and contribution of a young stellar component to the light flux, turn–off point (TP) of the main sequence, mass-to-luminosity ratio of the apparent stellar component of the galaxy. Using a population synthesis package of programs "SYNTHES" calculations have been performed for two normal galaxies, NGC 205 and NGC 628. Further possibilities and prospects of the proposed technique for population modelling are discussed.

Key words: galaxies:modelling – normal galaxies: population synthesis

1. Introduction. Physical formulation of the problem

The task of obtaining quantitative information on stellar population of composed systems has been considered by a number of authors using different techniques, each of which has both merits and demerits. Two principal directions in investigations of this kind are: evolution synthesis and population synthesis methods.

The method of evolution synthesis is based on the knowledge of processes of stellar evolution and employs a set of calculated isochrones for stars of different masses. The model spectrum corresponding to the observed one is created from a set of reference groups of stellar population having a definite age, chemical composition and star–formation rate. In principal this method can be used to get ample information, however the presence of a great number of free parameters (initial mass function (IMF), star–formation rate, set of ages and metallicities of stellar groups etc.) causes considerable uncertainties in the results. In addition the method is burdened with all errors resulting from the current concept of stellar evolution, the primary of which is the uncertainty in allowance for the contribution of stars at the late stages of evolution of giants. Analysis of stellar populations by the evolution synthesis method has been considered in numerous papers, Bica (1988), Bica et al. (1990), Gunn et al. (1981), Arimoto and Yoshii (1987), etc.

The technique of population synthesis in its different modifications (methods of linear and square programming with the application of a variety of constraints and conditions, parametric models (Williams, 1976; Faber, 1972; O'Connell, 1976) has been successfully used in the study of stellar composition of elliptical galaxies, early–type galaxies and nuclei of giant spiral galaxies, and under some additional conditions it allows the detection and separation of a nonstellar component in the spectrum of an object being studied. The synthetic spectrum of the system thus obtained can be used to investigate the history of evolution, for instance, the luminosity function, the relation between different age stellar groups, the presence and the volume of a young stellar component.

One of the important characteristics of the synthesis method is the selection of a suitable library of reference spectra of which a model spectrum of the galaxy is made up. In a number of papers (O'Connell, 1980; Spinrad and Taylor, 1971) a bank of individual reference stars of the wide range of spectral and luminosity classes is used. In some cases spectra of stellar mixes and globular clusters (Williams, 1976) are added to the stellar library. There is also another approach to the creation of such a library: the use of exclusively integral spectra of star clusters with various characteristics (mainly a set of ages and metallicities (Bica and Alloin, 1986)), which can either be taken from observations or be theoretical, as the constituents of the bank. The choice of an appropriate database, using the reference spectra depends on the problem formulated: in which form it is desirable to obtain information on the stellar population of the galaxy and its physical characteristics.

The objective of the present paper is to create a system for detailed determination of stellar population composition of complex stellar systems (galaxies, star clusters, etc.) using the population synthesis method. The technique of empirical population synthesis deconvolves a spectrum into components of the population vector whose basis is the library of light fluxes of reference observational spectra of stars and star clusters. In this method there are no constraints as to the kind of library to be used: it may consist of reference star spectra obtained in observations or theoretical spectra; or of observational or theoretical spectra of star clusters with different characteristics.

Parameters that might be obtained here include: the relative content of stars of different spectral types and of their luminosities, the average metallicity of a composite stellar system, the main sequence (MS) turn–off point, and therefore the age of the last significant star formation in the system and the contribution from the addition young stellar component, and also the mass–to–luminosity ratio ($\mathcal{M}/L_v$) of the visible component of the galaxy, which has an advantage of being independent of distance.

This paper presents basic propositions and a detailed description of the package of programs "SYNTHES" based on the population synthesis method (Section 3), requirements placed on the database for the task being considered (Sections 2,5), procedure of determination of astrophysical parameters of an object under study (Section 4).

2. Database: banks of reference stellar spectra

For the procedure of population synthesis of galaxies to be advantageously realized, a comprehensive spectrophotometric library is needed, which could cover the whole range of spectral and luminosity classes on the Herzsprung–Russel (H–R) diagram as fully as possible. Such a database, a bank of reference stellar spectra (BRS), is an important tool in the technique of spectral synthesis. For population synthesis the completeness of BRS is a crucial factor since in the process of synthesis it is impossible to define or compensate for a desired stellar component of the galactic population missing from BRS.

Besides, an important requirement imposed on reference spectra is as large wavelength coverage in the long–wave region as possible for more accurate determination of flux contributions from the late spectral classes M (for luminosity classes V and III). It is also desirable to include in BRS stellar groups with essentially different metallicities. This will provide an opportunity to derive the average metallicity for the system as a whole and also to draw a conclusion concerning chemical composition of its particular stellar populations.

To solve the problems of population synthesis, two banks of reference stellar spectra have been selected, which meet chiefly the requirements mentioned above. These are BRS of Pickles (1985a) and BRS of Jacoby et al. (1984).

2.1. Pickles' bank of stellar spectra

The original BRS of Pickles, the author has kindly made available to us, consists of stellar spectra in the wavelength range 3600–10000Å with a resolution of 10Å and spectral data increment of 3Å. The accuracy of the spectrophotometric data of the bank is about 1%. The spectra of stellar groups are presented in relative values of the flux F_λ normalized to 100 within the wavelength range 5450–5500Å. Pickles' bank is based on the spectra of about 200 objects, embracing the parts of the H–R diagram which are most essential for population synthesis of basically evolved, mixed, as to their population and chemical composition, stellar systems.

All in all the bank contains 48 types of stellar spectra, including the main–sequence standard groups of spectral classes (Sp) from OV to M5–6V, three groups of subgiants of class GIV and a branch of giants (III) extending from Sp B to M6. One spectral class comprises from two to four spectral groups.

A distinctive feature of the bank is that it contains three standard sequences of G–K giants specially formed for it. These giants have essentially different metallicity: metal–rich (mr) ([Fe/H] = $0.2 \div 0.4$) giants of solar composition, and metal–weak (wk) ([Fe/H] = $-1.6 \div -2.4$). The groups of blue giants A7–F0 of solar composition A7–F0 and late–F as well as wkF0 giants and the metal–weak group HB are classified as horizontal branch (HB) giants on the H–R diagram.

Prior to being used in the package of programs "SYNTHES" the original bank of Pickles was somewhat transformed: the initial stellar groups of BRS were recombined by selection of similar spectra and averaging of them in the new groups. The necessity of this transformation is caused by the fact that the observational data on the spectra of galaxies we have available have a noise level less than 5%. Therefore, in the case of using the original bank of Pickles for synthesis of such a spectrum, the likeness of the spectra of the neighbouring stellar groups of BRS results in large errors. For the transformation of Pickles' original BRS the latter was examined with the aid of correlation functions to reveal closely spaced multidimensional vector–spectra of the stellar groups of the bank. The adjacent reference spectra were selected and averaged if their correlation coefficient turned out to be larger than 0.90.

The values of the combined groups of Pickles' BRS (color index (B–V), absolute magnitude M_v, stellar group mass (in $\lg\mathcal{M}/\mathcal{M}\odot$), as well as metallicities for mr and wk giants) needed for calculations of the characteristics were obtained from the corresponding parameters of Pickles (1985a) by linear interpolation.

Thus, as a result of combining the groups of reference spectra of Pickles' original BRS, a modified version of the bank has been obtained, which contains 32 groups of reference spectra with a 3Å step in the wavelength region 3600–10000Å and a noise level of about 1%.

2.2. Jacoby's bank of stellar spectra

The library of reference stellar spectra of Jacoby consists of spectra covering the wavelength range 3510 – 7427Å for 161 stars of spectral classes O–M and luminosity classes V, III and I, having solar metallicity.

The reference spectra are given with 1.4Å step, their resolution is about 4.5Å. The photometric error for each spectrum element is about 1%, wide–band variations are less than ≈3%. The stellar spectra are given in terms of the flux F_λ.

To make Jacoby's library applicable in the program package "SYNTHES", a number of transformations of its stellar spectra were accomplished. Firstly, the bank spectra were re–binned using a spline method to the wavelength scale, adopted in the program: with the initial point at $\lambda = 3600$Å and a ≈ 3Å step. This should be done to allow comparison of the results of synthesis performed with the different banks (Pickles' and Jacoby's). Secondly, the absolute flux values in each spectrum of Jacoby's bank are normalized to the flux in a given stellar spectrum in the wavelength region 5450–5500Å, whose value is adopted to be 100 (i.e. fluxes are reduced to the standard of Pickles' BRS). Thirdly, similar to the procedure made with Pickles' BRS the groups of Jacoby's BRS were re–combined and enlarged using the same principle. An additional condition needed when grouping the spectra of the bank was such a choice of membership of the groups (by spectral classes) that they fit best to their associated break down into groups of Pickles' BRS.

The quantities (B–V) for each group of the bank were defined from (B–V) for the individual stellar spectra contained in the group of data in Jacoby et al. (1984) by linear interpolation; the absolute magnitudes M_v for the groups of the bank were also interpolated from the relationship M_v–Sp presented by Pickles (1985a); the values of $\lg \mathcal{M}/\mathcal{M}_\odot$ were calculated in a similar manner from the combined relations $\lg \mathcal{M}/\mathcal{M}_\odot$–Sp from Pickles (1985a) and Tinsley (1980).

Thus, the bank of reference stellar spectra adopted to our objective has the following characteristics: it contains 36 groups of spectra, which represent spectral classes from O to M and luminosity classes V, III and I with solar metallicity in the wavelength range 3600–7427Å with a 3Å step and photometric accuracy of about 1%.

2.3. Comparison and selection of BRS for population synthesis

A number of inferences can be made when comparing the two BRS:

- The overwhelming majority of spectral groups restricted in spectral classes are identical for both banks, the agreement in the value of relative intensities $100 \cdot F_\lambda/F_{\lambda_0}$ is good enough, differences do not exceed 5%.
- Some spectra of hot stellar groups (MS and giants), for which a steeper rise in the Jacoby's bank spectra is observed in the region of the Balmer discontinuity, are an exception. For this reason the discrepancy in relative intensities for the spectral groups BV and B–A–F0 III in a small wavelength region, 3600-4100Å, may reach 15%.
- An essential difference between the two banks being compared can be revealed when examining intensities and shapes of absorption lines in their spectra. The lines in Jacoby's bank are narrower and deeper than those in the spectra of Pickles' bank. The discrepancy in depths at the line center amounts to 1.5–2 times. The smoothness of the spectral line profiles in Pickles' BRS as compared with Jacoby's is due to the difference in resolution of the spectrophotometric data of the banks.

To select an appropriate library of reference stellar spectra for population synthesis, one should take into account completeness of the set of stellar populations represented in it. From this point of view Pickles' BRS has the advantage that it contains stellar groups of giants of essentially different metallicity ([Fe/H] from 0.4 to −2.4), which is important for synthesis of spectra of evolved stellar systems with presumably different chemical composition of populations comprising them. However there are no hot young supergiants (luminosity class I) in this bank.

On the contrary, Jacoby's BRS, which includes supergiants of class I apart from classes V, IV and III, is more advantageous to be used for synthesis of young stellar systems with high–rate star formation, where the young stellar component is the main contributor to the spectrum. At the same time in this case it seems impossible to stratify population groups in the galaxy by different metallicities, because all the reference groups of the bank have solar metallicity. That is why the attempt of synthesizing the spectra of the galaxy with a presumably great spread in metallicities of the stellar groups it consists of, may lead to erroneous results.

Now, a conclusion can be drawn that for population synthesis it is possible to use both BRSs, taking into account the type of the object under investigation. The choice between them must be made proceeding from the characteristics and parameters of the object (synthetic spectrum, population stellar composition, metallicity, age, turn–off point, etc.) which are expected to be obtained.

3. Population synthesis process

3.1. Mathematical statement of the problem. Selection of technique

The problem of population synthesis is solved in the present paper with the algorithm described by Forsythe et al. (1980). This algorithm relies on the well known mathematical class of problems of numerical solution of the minimizing problem by the least squares method (LSM) and is based on the technique of singular decomposition. One of the principal difficulties encountered in population synthesis is the solution non–singularity.

In terms of mathematics it means that there is strong correlation between the vectors of the reference stellar spectra forming the base of BRS, into which the decomposition is made. Hence the mathematical problem of synthesis is a degenerate one. At the same time inevitable intrinsic errors in the observed spectra of the galaxy as well as difficulties in construction of the BRS base, which includes all possible elements of the stellar and non–stellar components of the galaxy population, also leads to possible existence of several versions of solution, depending on the adopted accuracy or on the particular choice of the BRS base.

Besides, the problem solution is affected by the choice of mathematical method: linearity or nonlinearity of the function that is minimized and the presence of some astrophysical constraints of the parameters of the equation set. In an ideal case, when the spectrum library base is complete and there is no noise, the local minimum of the discrepancy function is single in the vector space. Then the only correct solution is independent of the initial conditions, assumptions and constraints. As for any real synthesis, the local minimum of the discrepancy function is a particular region, where the discrepancy is minimum within the uncertainties of observations and accuracy of synthesis.

The main factors that affect the size of the region within which the solution converges are: a) completeness of the library; b) observational accuracy of the galactic and library spectra.

Applying different approaches to the selection of BRS for decomposition of the galactic spectra and solving the problem of synthesis with different degree of approximation accuracy, one can obtain a set of solutions whose acceptability must be evaluated from the astrophysical point of view. In a number of papers (O'Connell, 1976; Pickles, 1985b) the authors confine themselves to solutions satisfying the knowledge of stellar evolution by means of imposing limitations on the sought–for contributions of fluxes of stars as a priori relations between numbers of stars of different types. These limitations often have the form of detailed quantitative relations between different groups for the MS (which actually predetermines the slope of the luminosity function) or the form of a priori specified relations between the number of stars of different luminosity classes.

Such artificial specification of relations of flux contributions from different spectral groups to the integral spectrum undoubtedly restricts the number of the solutions selected for synthesis of the galactic spectrum and guarantees an astrophysically plausible distribution of the number of stars. However this does not mean at all that in such a manner the true stellar mix in the galaxy will be find, especially if its stellar population has been formed as a result of several star formation bursts. In this case the relationship between the stellar types may turn out to be not as trivial as has been anticipated. The setting of rigid relations between the MS groups levels the possible presence of peaks in the distribution caused by star formation bursts.

The rejection of the a priori limitations on the distribution of stellar population in the galaxy in our technique causes the problem considered to become linear with respect to the sought–for coefficients of light contributions, and one can use the least squares method to solve it.

The use of singular analysis to solve the given problem allows full analysis of stability for the particular base (bank of reference spectra) employed and, accordingly, introduction of appropriate corrections into the selected bank of reference spectra. The negative coefficients of flux contributions, which appear occasionally in the solution, are exactly the indicators of the direction of needed BRS changing. The presence of such coefficients in the solution implies that their associated stellar groups of BRS are missing from the stellar population of the object under investigation or BRS is not complete. Proceeding from this, the selected technique permits us to get rid of possible negative coefficients and pick up a suitable solution to the problem of synthesis without specifying preliminarily any complementary relations between the sought–for coefficients.

3.2. Population synthesis algorithm

The problem of population synthesis defined in this paper consists in sampling astrophysically acceptable spectra of the BRS stellar groups, which would form a galactic spectrum most similar to the observed one. The search for the solution is made by way of minimizing the discrepancy between observed and computed energy distribution in the spectrum.

Mathematically the problem is formulated as follows: the desired parameters of flux contributions over the group of stars C_j (j=1,..., n) are determined as a result of minimizing the sum of squares of local discrepancies:

$$q = \sum_{i=1}^{m} r_i^2 = \sum_{i=1}^{m} \left[g_i \quad - \sum_{j=1}^{n} C_j \cdot f_{ij} \right]^2 , \qquad (1)$$

where r_i is the local discrepancy found as the normalized difference in fluxes between synthetic and observed spectra; $g_i = 100 \cdot G_i / G_{\lambda_0}$, the normalized flux from the observed galaxy (G_i, the observed flux at the i–th point of the spectrum (i=1,...,m); G_{λ_0}, the observed flux for the wavelength 5450Å); $C_j = (100 \cdot N/R^2) \cdot (F_{\lambda_{0j}}/G_{\lambda_0}) \cdot X_j$, the contribution to the normalized flux of the galactic spectrum from the j–th stellar group of BRS at λ_0 (N, the total number of stars in the galaxy; R, the distance to the galaxy; X_j, the fraction of stars of the j–th group in the galaxy). All contributions C_j are expressed in fractions of unit so that $\sum_{j=1}^{n} C_j$=1); $f_{ij} = 100 \cdot F_{ij}/F_{\lambda_0 j}$ is the normalized flux from the j–th spectral group (F_{ij}, the flux from a star of group j (j=1,...,n) from BRS at the i–th point of the spectrum; $F_{\lambda_{0j}}$, the flux from a star of the j–th group at $\lambda = 5450$Å).

In general, the tactics of search for optimum solution of the set of linear equations is as follows. First, the LS problem for the complete bank is solved using a selected algorithmic scheme, and the flux contribution coefficients, the error of the contribution coefficients values, the local and total discrepancies are computed as well as the mean discrepancy value, determined by the formula:

$$q_{mid} = \sqrt{\left(\sum_{i=1}^{m} r_i^2 \right) \Big/ \ (m - n)}, \qquad (2)$$

where m is the number of points along wavelength in stellar and galactic spectra, n is the number of stellar groups.

Such a calculation is done in turn for each of the available spectrum of the galaxy, the results are averaged. After that, the procedure of selection of the sample of stellar spectra from BRS for the main sequence, which are really present in the stellar population of the galaxy, is performed. Commencing from the complete sample of the main–sequence stars the first (second, etc.) group of stars is sequentially excluded, i.e. the presumed turn–off point is moved to the next stellar group along the main sequence. At each step of this cycle the discrepancy value is analyzed. Over the whole collection of values a search for minimum discrepancy is made, and then the corresponding set of stellar groups of the MS to be synthesized is defined.

Thus, as a result of the described procedure of sampling BRS elements, we obtain the vector C_j ($j = 1, \ldots, n$) of flux contributions to the synthetic spectrum of the galaxy from the BRS stellar groups, which in the aggregate comprise the stellar population of the galaxy.

3.3. Analysis of calculation errors

In the singular–analysis method applied to the linear LS problem of population synthesis there is an important parameter, which affects considerably the reliability of solution. This is the specified accuracy of the synthesized spectrum approximation to the observed one. To solve the problem of synthesis and obtain an astrophysically correct result, it is important to properly select this parameter. Its value must be consistent with the accuracy of the initial data (data errors in BRS and observed spectra).

From the physical point of view it is clear that the degree of similarity of the synthetic spectrum to the observed one should not be set higher than the relative error in the energy flux of the galactic spectrum for which synthesis is performed. Otherwise, although the solution shall be found, it will be unacceptable from the astrophysical point of view. It should be pointed out too that the solution may be regarded applicable if not only a sufficient degree of similarity of the synthesized spectrum of the galaxy to the observed one is attained (i.e. the discrepancy is small), but also the errors in determining flux contributions of the stellar groups from BRS, which constitute this spectrum, are within reasonable limits. The error in determining each C_j element is different and depends on the corresponding spectrum of the stellar group of BRS, varying within one order. The errors in determining the components of the vector $\vec{C}$ are calculated with the covariation matrix of the LS problem solution. In a singular decomposition the diagonal elements of the covariation matrix are the values of determination errors of the associated vector $\vec{C}$ and determined by the formula from Lowson and Henson (1986):

$$cov_{jj} = \sum_{i=1}^{k} v_{ji}^2 / \sigma_{ii}^2, \ i = 1, \ldots, n, \qquad (3)$$

where cov_{jj} are the diagonal elements of the covariation matrix; v_{ji}^2 are the elements of the orthogonal matrix V from the singular decomposition of the matrix $A = U \cdot \sum \cdot V^T$ for the set of linear equations $A \cdot x = b$ (A, the matrix $m \times n$, $m > n$; m, the total number of observations; n, the total number of variables; b, the vector of dimensionality m; $U(m \times m)$ and $V(n \times n)$ are the orthogonal matrices; $\sum$ is the diagonal matrix (Forsythe et al., 1980); σ_{ii}, the singular numbers or elements of the diagonal matrix $\sum$; k, the number of singular numbers, which are taken into account when calculating the elements of the vector $\vec{C}$.

The calculations have shown that the range of individual errors of contribution may essentially differ for different groups of stars. The larger error of the flux contribution for some groups of BRS is due to the presence of one or several stellar spectra from BRS groups, which are strongly correlated with the one under consideration.

Thus, the accuracy of determination of the vector components depends not only on the noise level but also on the set of reference spectra from BRS. It has been shown in the calculations that the less reliable determinations are more frequent for the groups comprising early blue spectral classes A0–3 V; B III, HB and late spectral classes M5–6 III, as compared to the rest of the groups, and less frequent for the groups of spectral classes M4–6 V.

Cases are possible when the flux contribution of some component is small and negative, while the error value is comparable with the contribution itself. Then within the calculation such a component C_j may be assumed equal to zero. For control over the situation of this kind the reliability degree of determination of each component of the vector $\vec{C}$ is calculated: $d_j = C_j / cov_{jj}$, $j = 1, \ldots, n$. The component C_j is considered determined reliably with a probability of 99% if at n=20–30 $d_j < 2.75 \div 2.85$ (Shchigolev, 1969).

4. Determination of astrophysical parameters of extragalactic objects from their integral spectra

4.1. Turn–off point

The first astrophysical parameter determined in the process of synthesis is the location of the turn–off point of the MS for the stellar population of the galaxy. In terms of our model it means that it is necessary to find the appropriate set of stellar groups from BRS, which make up the MS of the given galaxy; it is the first of the stellar groups of the upper end of the MS that will define the turn–off point.

In the case of test model calculation the spectrum of the galaxy is known quite accurately, then synthesis with the full MS from BRS gives zero contributions for the stellar groups lacking in the galaxy (because the technique does not introduce calculation errors into the solution). However, the real spectrum is always noisy, therefore contributions of "excess" groups will be nonzero since they have some scatter due to the errors of determination of flux contributions. For separation and rejection of such groups at the upper end of the MS the method of search for minimum discrepancy over the whole collection of solutions when varying the set of the MS from BRS is used. Here the following idea is applied: when BRS is redetermined, the average discrepancy will be larger than with the true set of the MS stellar groups for the given galaxy.

Analysing the discrepancy behaviour when modelling the spectrum of the object being investigated, one can find both the turn–off point for the youngest group of the stellar population of the galaxy (limiting point of the upper end of the MS) and the TP for the old (most numerous) population of the galaxy. It falls at the later spectral classes of the MS and therefore does not show up explicitly in the distribution of light contributions from the aggregate galactic stellar mix of different periods of star formation.

Three versions of discrepancy behaviour are mainly possible, depending on the stellar composition of the galaxy. If the young stellar population is the major contributor to the light of the galaxy, the discrepancy has only one minimum falling at the early stellar groups of the MS. That is, the further sequential exclusion of stellar groups from the blue end of the MS from the spectrum modelling shall increase the discrepancy. There is another extreme case, when the object is represented exclusively by the old stellar population (for instance, a globular cluster). Here the curve of discrepancy variation has also only one minimum, depending on the adopted length of the MS, but the minimum is in the region of spectral classes corresponding to the TP of the most numerous initial population. In this case bluer stars, when added to the MS, affect the solution. Based on this, we can conclude that there are no blue stars of luminosity class V in such a stellar population.

As a rule, the stellar population of real galaxies with complex stellar composition is an intermediate variant between the two considered. That is, they have a characteristic discrepancy minimum due to the presence of a portion of light from the old stellar population in the galactic spectrum. However, this minimum is local since the involvement in synthesis of stellar groups of the blue part of the MS improves the solution. Then the turn–off point on the MS for the young stellar component of the galaxy is defined from the global minimum on the plot of discrepancies, while that for the old stellar population is estimated from the secondary local minimum.

4.2. Metallicity of stellar populations

Pickles' BRS comprises the groups of giants with different chemical composition, for which their metallicities are presented. Therefore, when used as the data base in synthesis of the galactic spectrum it enables one to estimate the percentage of the galactic stellar populations with different chemical composition. Having computed flux contributions for these groups of giants we can obtain the percentage relation (by flux contribution to the spectrum and by mass) between the groups and the weighted mean metallicity for each of them.

The flux contribution for each of the three groups of giants (with respect to the total flux contribution of all the giants) is equal to:

$$\Delta C_g = \frac{\sum_j C_{gj}}{\sum_g \left(\sum_j C_{gj}\right)}, \tag{4}$$

and the mass fraction calculated in a similar manner:

$$\Delta \mathcal{M}_g = \frac{\sum_j m_{gj} \cdot N_{gj}}{\sum_g \left(\sum_j m_{gj} \cdot N_{gj}\right)}, \tag{5}$$

where g is the index designating the type of the group of giants (mr, high metallicity; sun, solar composition; wk, low metallicity); C_{gj} is the flux contribution of giant stars of the j–th type from group g; m_{gj}, the mass of giant stars of the j–th type; N_{gj}, the number of the j–th type giants from group g. The summation is done over the spectral types of the considered group of the giants which are present in the galaxy being simulated.

The weighted mean abundance of elements by flux contribution (similarly by mass) for the group of giants is calculated by the formula:

$$Z_g = \frac{\sum_j C_{gj} \cdot Z_{gj}}{\sum_j C_{gj}}, \tag{6}$$

where Z_{gj} is the metal abundance in giant stars of the j–th type from group g; solar abundance $Z_{sun} = 0.017$.

The weighted mean metallicity by flux contribution (similarly by mass) of the whole group of giants in the galaxy under study can be calculated by the formula:

$$[Fe/H]_{mid} = \frac{\left(\sum_g \Delta C_g \cdot FH_g\right)}{\left(\sum_g \Delta C_g\right)} - 3, \tag{7}$$

where $FH_g = [Fe/H]_g + 3$. The intermediate shift of the logarithmic scale of metallicities by 3 units is needed to make it positive; it turns out possible then to allow for the contribution to $[Fe/H]_{mid}$ of solar metallicity giants too, for which $[Fe/H]_{mid} = 0.0$.

4.3. Age of stellar groups

In the process of synthesis of the galactic spectrum the turn-off point of the MS for the main, old stellar population of the galaxy is found. In most cases this population is the major contributor to the flux from the stars on MS. In the case of an appreciable contribution from the blue stellar component of the MS to the radiation of the galaxy, it is possible to define the turn–off point for the young stellar population of the MS.

Next, from both TP of the young and old components of the stellar population we derive the values of the colour indices (B–V)$_{TP}$ for these portions of the MS (which are presented in Pickles and Jacoby for the corresponding groups of BRS). Metallicity is the second parameter employed in the determination of the age of stellar groups. So, it is necessary to consider which metallicity from the Z values determined in Section 4.2 and which components of stellar population of different age are associated.

Metallicity of the MS stars near the turn–off point for the old stellar population of the galaxy will probably be consistent with the weighted mean metallicity of mr giant stars (4.2). Indeed, proceeding from the theory of stellar evolution, the objects that populate the giant branch and horizontal branch are chiefly evolved stars of about the same age and chemical composition as the initial population of the galaxy, which account for the lower part of the MS. However, metallicity of stars of the turn–off point of the young stellar population may probably be estimated to be close to that of mr giants or, if there are no such giants in the stellar mix, the chemical composition of this part of the population should be considered equal to solar.

The age of each of the stellar components of the galaxy, for which (B–V)$_{TP}$ and Z have been determined, are computed with the help of isochrones of Demarque et al. (1987). The calculation is done through linear interpolation from two parameters from the table of isochrones within: (B–V)$_{TP}$: $-0.25 \div +1.15$; Z: $0.00001 \div 0.10$; and age $10^8 \div 3 \cdot 10^{10}$ years. The helium abundance is taken equal to $y = 0.25$.

4.4. Analysis of mass function

The results of population synthesis of the galactic spectrum allow the calculation of the mass function of the galactic stellar populations. The algorithm of such a calculation is as follows. The quantities C_j obtained from the synthesis, mean the flux contribution of stars of the j–th group to the spectrum of the galaxy at the standard wavelength $\lambda = 5450$A. Having the distribution of flux contributions, one can compute the distribution of the number of stars over the groups:

$$\lg N_j = \lg C_j + (M_{sj} - M_g)/2.5, j = 1, \ldots, n. \quad (8)$$

The formula has been obtained taking into account the relation $C_j \cdot F_g = N_j \cdot F_{sj}$, where N_j is the number of stars of the j–th group; F_{sj} is the flux of the j–th group at $\lambda = 5450$Å; F_j is the light flux from the galaxy at $\lambda = 5450$Å; M_g is the absolute V magnitude of the galaxy; M_{sj} is the absolute V magnitude of the j–th group of the reference stars.

Next, assume that the distribution of the number of the main–sequence stars by masses m_j occurs by a power law whose parameters are different for the upper and lower part of the main sequence:

$$\begin{cases} N_{(m_j)} & = & P_1 \cdot m_j^{-x_1} & m \geq m_t \\ N_j(m_j) & = & P_2 \cdot m_j^{-x_2} & m \leq m_t. \end{cases} \quad (9)$$

Here x_1 and x_2 are the slopes of the upper and lower part of the main sequence, respectively; P_1 and P_2 are the corresponding free coefficients for the upper and lower part of the mass function (MF) curve.

When passing to the logarithmic scale, the equations become linear with respect to the parameter x and can be solved by the least–squares method. The m_t limit on the mass scale is determined from the minimum of the total discrepancies for both straight lines, that is, square deviations of the graph points from the corresponding approximating straight lines for the given part of the distribution.

4.5. Parameter $\mathcal{M}/L_v$ and population of MV stars

The parameter $\mathcal{M}/L_v$ characterizes directly the ratio of the number of low–mass dwarf stars which make up the dominant share of the total mass, to the number of massive stars, whose contribution to the integral galactic spectrum is most appreciable and defines frequently its form.

The ratio of the luminous mass of the galaxy to its luminosity is deduced from the formula:

$$\mathcal{M}/L_v = \sum_{j=1}^{n} m_j \cdot N_j / \sum_{j=1}^{n} V_{sj} \cdot N_j. \quad (10)$$

Here $\mathcal{M}$ and L_v are expressed in solar units. Then the flux from the galaxy at λ=5450Å also needs to be normalized to the flux from the Sun. Therefore, the fluxes from the stellar groups which constitute the galactic spectrum are determined as $V_{sj} = F_{sj}/F_{sun}$, where F_{sj} is the flux from the j–th type stellar group, while F_{sun} is the flux from the Sun. The fluxes V_{sj} are computed through the values of the absolute V magnitudes M_{sj} of each spectral group from the formula:

$$V_{sj} = 10^{-0.4(M_{sj} - M_{sun})}, \quad (11)$$

where M_{sun} is the absolute magnitude of the Sun.

The computations show that the number of cool M0–6 V dwarf stars of the MS is, as a rule, by 0.5–1.0 order larger than that of the stellar groups of the earlier spectral classes. This is accounted for by the sharp drop in the luminosity of the main–sequence stars, beginning with spectral class M0, with the much slower decrease in their mass. In turn, this leads to the fact that the main contribution to the total mass of a stellar object comes from cool dwarfs of class M. Note, that the error in determination of C_j for the red dwarfs M0–6 V defines the error in determination of the parameter $\mathcal{M}/L_v$.

5. Application of population synthesis technique to the galaxies NGC 205 and NGC 628

To test and debug the package of programs "SYNTHES", spectra of the normal galaxies, NGC 205 and NGC 628, with considerably different physical characteristics, were used. The spectra were obtained on the 6 m telescope of SAO RAS at the Nasmyth–1 focus with a 1000–channel scanner. All in all 4 spectra of NGC 205 and 12 spectra of NGC 628 were obtained in two partially overlapping spectral regions: blue 3600–5500Å and red 53200–7200Å. The primary reduction of the spectra was done by the standard methods (Somov, 1986; Afanasiev et al., 1991).

The accuracy and resolution of observational data are the following: noise level $> 5\%$, spectral resolution 4.5Å, after the reduction and smoothing the data are tabulated in steps of $\lambda \approx 2.5 \div 3.0$Å.

The next two sections present optimum models for the two galaxies and results of determination of their astrophysical parameters.

5.1. Synthesis results for the galaxy NGC 205

NGC 205, a satellite of M 31, is a dwarf elliptical galaxy, which, however, may turn out to be a spiral one: faint spiral arms starting at the opposite sides of the galaxy have been detected. So, NGC 205 may be regarded as a barred galaxy with a powerful bar. It is of spectral class A8, which is unusually early for elliptical galaxies. Some anomalies are observed in the spectrum. In the central region of the galaxy blue OB stars of high luminosity have been revealed. All these data allow the assumption that this object is of complex stellar composition.

The number of spectra used in the synthesis is 4; the effective accuracy of the original spectra of the object was taken to be $P \approx 5\%$. The minimum discrepancy is $q = 6.32$. Spectral range is 3702÷7158Å. The spectra of NGC 205 were corrected for interstellar absorption (technique of Schild, 1977); here the amount of colour excess, E(B–V)=0.11, was taken from Bica et al. (1990). The synthesis of NGC 205 was

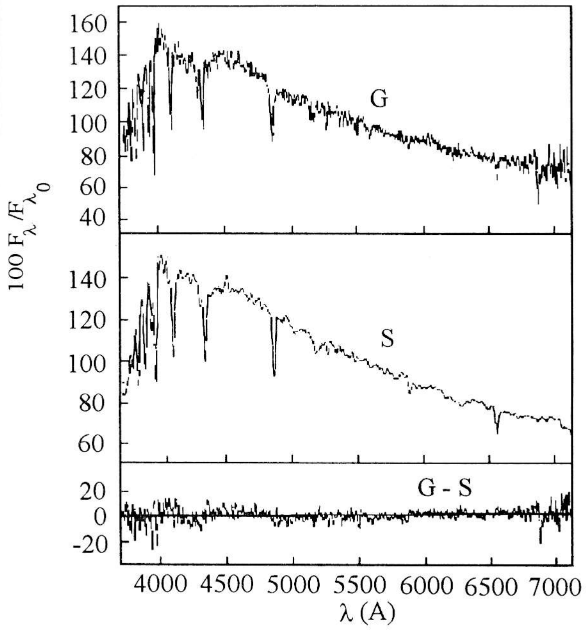


Figure 1: *Curve G — the observed averaged spectrum of the galaxy NGC 205. Curve S — the computed synthetic spectrum of NGC 205. Spectrum fluxes are normalized to 100 for the region V. Curve G–S — the residual difference between the observed and synthetic spectra of the galaxy.*

made with the modified bank of reference spectra of Pickles. In Fig.1 are displayed the observed (averaged over the 4 final spectra) spectrum of the galaxy, the computed synthetic spectrum and also the residual difference between them.

The model of NGC 205 is shown in Fig.2(a,b). Fig. 2a presents the percentage flux contribution distribution versus the reference stellar group which is present in the population model of the galaxy. The character of flux contribution distribution over the groups shows the presence of blue hot main–sequence stars (spectral class B4–A3), which give a considerable contribution, 16% of the integral flux (in the V band). The main sequence as a whole makes (in the V range) half of the total flux contribution to the integral light flux. Of this 49% more than 2/3 of the flux from the MS is produced by the young stars up to the turn–off point (F7–8), while the flux from the red part of the MS containing low–mass dwarfs shows up in the summary spectrum at the limit of errors. That is why the discrepancy secondary minimum at the turn–off point at Sp F7–8 is marked very poorly.

The flux contribution from the subgiants is insignificant, only 6.2%. The evolved stars are represented by the well developed horizontal branch of "solar" giants from B to F1 (19%) and by the low-metallicity group HB – wF0 (14%). The flux contribution of giants of different types of metallicity is insignificant and distributed as follows: the giants of solar chemical abundance account for 6%, *wk* giants with $Z = 0.00026 \pm 0.00001$ for 8%. Within the errors the flux contribution from the *mr* giants has not been detected.

Fig.2a presents the range of errors in determination of flux contribution to the summary spectrum from each of the spectral groups. The error for different groups is different and varies from $0.5 \div 1.0\%$ (as a percentage of the quantity under consideration) for well defined BRS groups to $20 \div 30\%$ for four groups: red dwarfs M0–6 V and red giants from K4–5 to M5–6. The great uncertainty in the calculation of flux contribution of their spectra is due to the insufficiently long red end of the simulated spectrum ($\lambda < 6780$Å). The mean error in determining flux contributions amounts to 2%.

From the synthesis results it can be seen that both the old stellar population, represented by the rich horizontal branch and low metallicity giants, and a considerable number of comparatively young blue stars in the upper part of the MS with solar chemical composition are present in NGC 205.

Fig.2b displays the distribution of the number of stars versus different groups of the stellar population of the galaxy and the errors of its determination. In columns 4–6 of Table 1 are listed the derived integral physical characteristics of NGC 205; summary luminous mass of the galaxy $\mathcal{M}$ (in solar masses), total number of stars N and mass–to–luminosity ratio parameter $\mathcal{M}/L_v$. In columns 2–3 are tabulated the distance values to the galaxy R and its apparent magnitude M_v which have been used in the computation (Buonanno et al., 1985). The obtained $\mathcal{M}/L_v = 2.3 \pm 1.0$ is in good agreement within the determination error with this parameter $\mathcal{M}/L_v$=2.35 derived by Carter and Sadler (1990) for the central region of NGC 205 from its kinematic characteristics.

Table 2 presents the results of determination of the following physical parameters of the galaxy: slope of the MF, chemical composition and age of its stellar populations. The slope of the MS in its upper and lower parts turned out to be practically the same but slightly higher than Salpeter's (IMF parameters, columns 2–3). Formula (7) was used to calculate the weighted mean metallicity values by flux, $[Fe/H]_{mid(l)}$ and by mass, $[Fe/H]_{mid(m)}$ (columns 4–5 of Table 2). The mean metallicity value of NGC 205, which is presented in Buonanno et al. (1985) is -0.85 ± 0.2. The age of the young stellar population (column 6 of Table 2) is defined from the location of the turn–off point for the last star formation burst (B4–9 V) and from the metallicity value $[Fe/H]_{mid(m)}$. The age of the old stellar component (column 7) can be estimated from the weak secondary discrepancy minimum (TP at F7–8 V) under the assumption that the original population of the

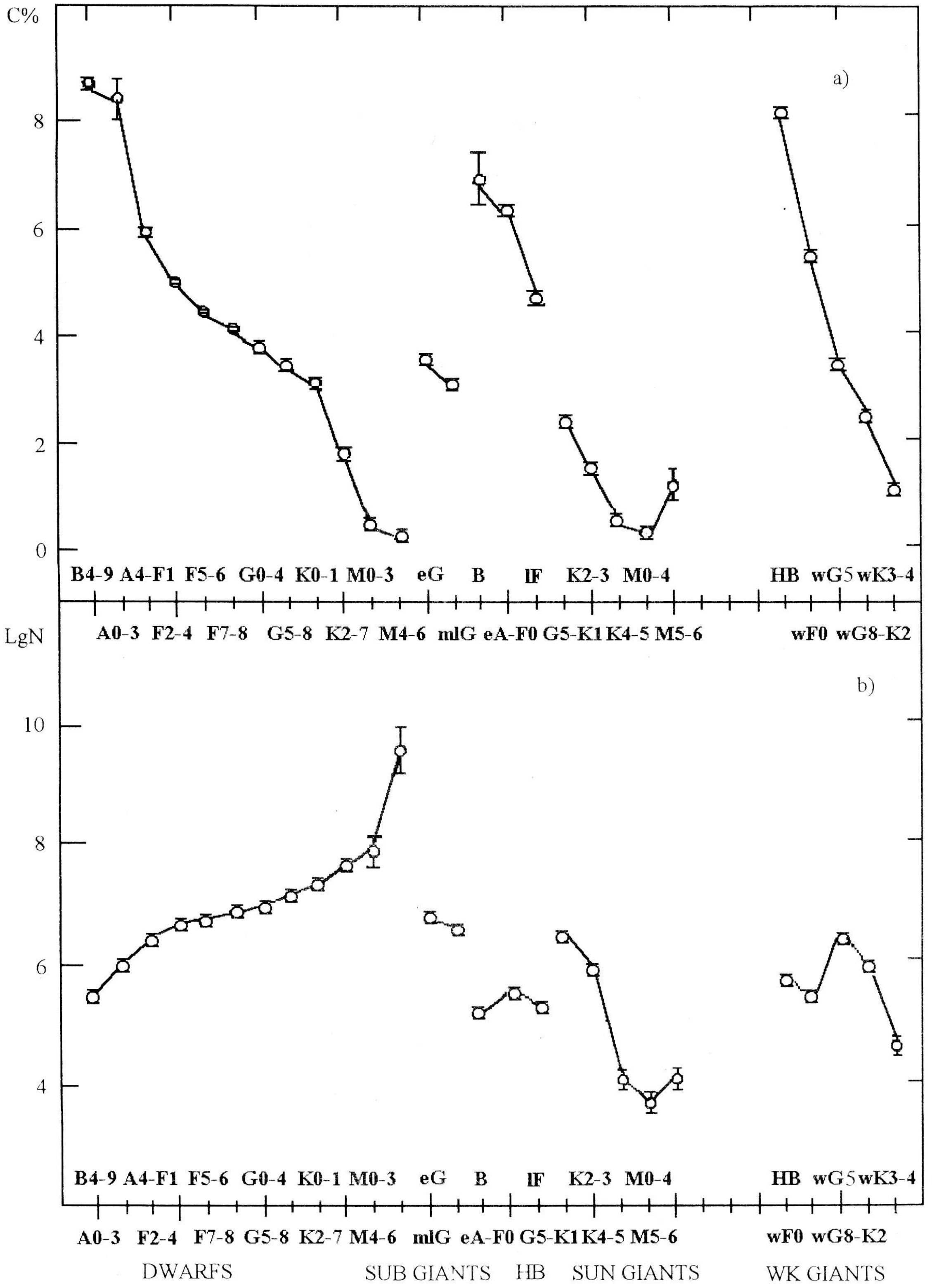


Figure 2: *The model for the galaxy NGC 205. a) The percentage distribution of flux contributions to the spectrum versus the BRS stellar group incorporated in the population model. b) The distribution of the number of stars over the groups of the stellar population of the galaxy. The abscissa is the spectral type for the BRS stellar groups and their luminosity classes. The thin vertical bars are the errors for each component of the vector* $\vec{C}$.

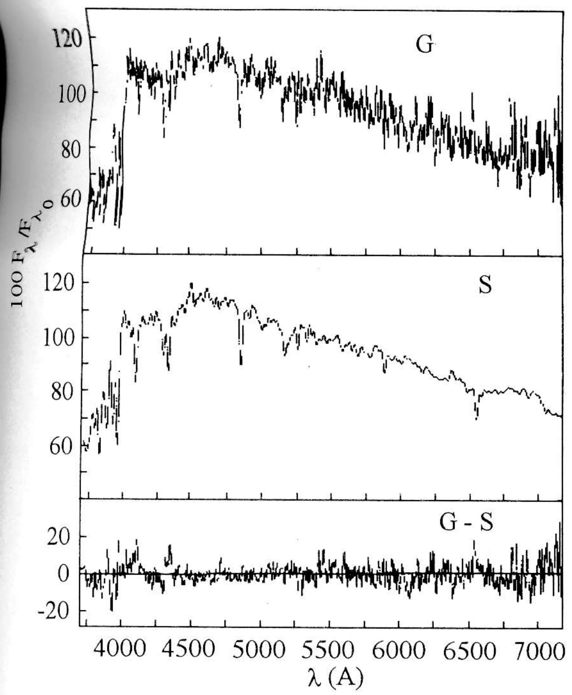


Figure 3: *The observed and synthetic spectra for the galaxy NGC 628 and the residual difference between them (designations the same as in Fig.1).*

galaxy has low metallicity definable by formula 4.2: $[Fe/H]_{wk} = -1.81 \pm 0.02$.

The population synthesis results were compared with the data obtained by Bica et al. (1990) for this galaxy by the evolution synthesis technique incorporating the bank of star clusters. From the data of Bica et al.(1990) the young population group of the galaxy NGC 205 contributes about 46% of flux to the summary spectrum and has the age of $10^8 \div 5 \cdot 10^8$ years. The age of the oldest stars of the galaxy is estimated by them to be more than 10^{10} years. From the data of the author the age of the young stellar population is $3.16 \cdot 10^8$ years and its contribution to the galactic spectrum is about 54%. The original stellar population has the age of about $(1.1 \pm 0.1) \cdot 10^{10}$ years. Thus, one can say that the results are in fair agreement.

5.2. Synthesis results for the galaxy NGC 628

NGC 628 (M74) is a giant spiral galaxy of type Sc with long and pronounced spiral arms in which numerous extensive star formation regions are observed. Its spectral class is F5.

Nine spectra were used for modelling; their effective accuracy is $P \approx 4\%$; minimum discrepancy is $q = 7.84$. The spectral range for modelling is 3696-7188Å. Synthesis was done using Pickles' BRS. Fig.3 displays the observed (averaged over the 9 spectra), synthetic spectra of NGC 628 and the residual difference between them.

The model of the galaxy is shown in Fig.4(a,b). The main sequence of the galaxy is represented by the aggregate of spectral classes, on which one can clearly see two groups of stellar population of different time of formation. This inference is confirmed by the plot of discrepancy dynamics (Fig.4c), from which the TP for the initial stellar population is defined in the region Sp F5–6, while the TP for the young stars of the last star formation burst in the region O8 – B1–3.

The flux from the MS forms half of the flux in the summary spectrum of the galaxy (44%), 2/3 of which is accounted for by the old primary population (late F stars and dwarfs of spectral classes G–K). An essential contribution to the flux from the MS is given by hot young O–B and A stars (4% and 10% of the total flux, respectively). 99.5% of all luminous mass of the galaxy is contained in the MS. The presence of a cosiderable share of subgiants (10% of the flux contribution) is observed.

A great contribution of the evolved stars (47% of the integral flux) is revealed. The most numerous of them is the horizontal branch (17% contribution) comprising "solar" giants B–1F and group HB–wF0 with low metallicity. The contributions of giants of different metallicity groups are distributed as follows: giants of solar composition account for 12%, mr–giants with $Z = 0.034 \pm 0.002$ for 10%, and wk–giants with $Z = 0.00024 \pm 0.00001$ for 8%. Attention should be paid to the presence of an appreciable share of red giants (M0–5 III) in the given model, whose flux contribution to the spectrum is by a factor of 4 larger (3.3%) than that of M dwarfs. The mean flux contribution determination error makes up 4%. Fig.4b shows the distribution of the number of stars versus the groups of the stellar population of the galaxy and the errors of its determination. In Table 1 are listed the obtained integral physical characteristics of NGC 628: mass $\mathcal{M}$, total number of stars N, mass-to-luminosity ratio $\mathcal{M}/L_v$.

The results of computation of the IMF slope for the MS of NGC 628 are presented in Fig.5b. The slope of the MS of the galaxy is different for the old and young components of the stellar population. Its upper blue part (up to Sp F5–6) has a steeper slope than the lower red part of the MS with dwarfs of the original stellar population. This is suggested by the mass function parameters from formula (9) (Table 2, columns 2–3). The computed weighted mean metallicity values by flux and by mass are also presented in Table 2 (columns 4–5). Finally, in the last two columns of Table 2 are given the ages of the young and old stellar components estimated from the turn-off point position. In the calculations for the young component we used the metallicity values obtained for mr–giants $[Fe/H]_{mr} = 0.30 \pm 0.03$, while for the

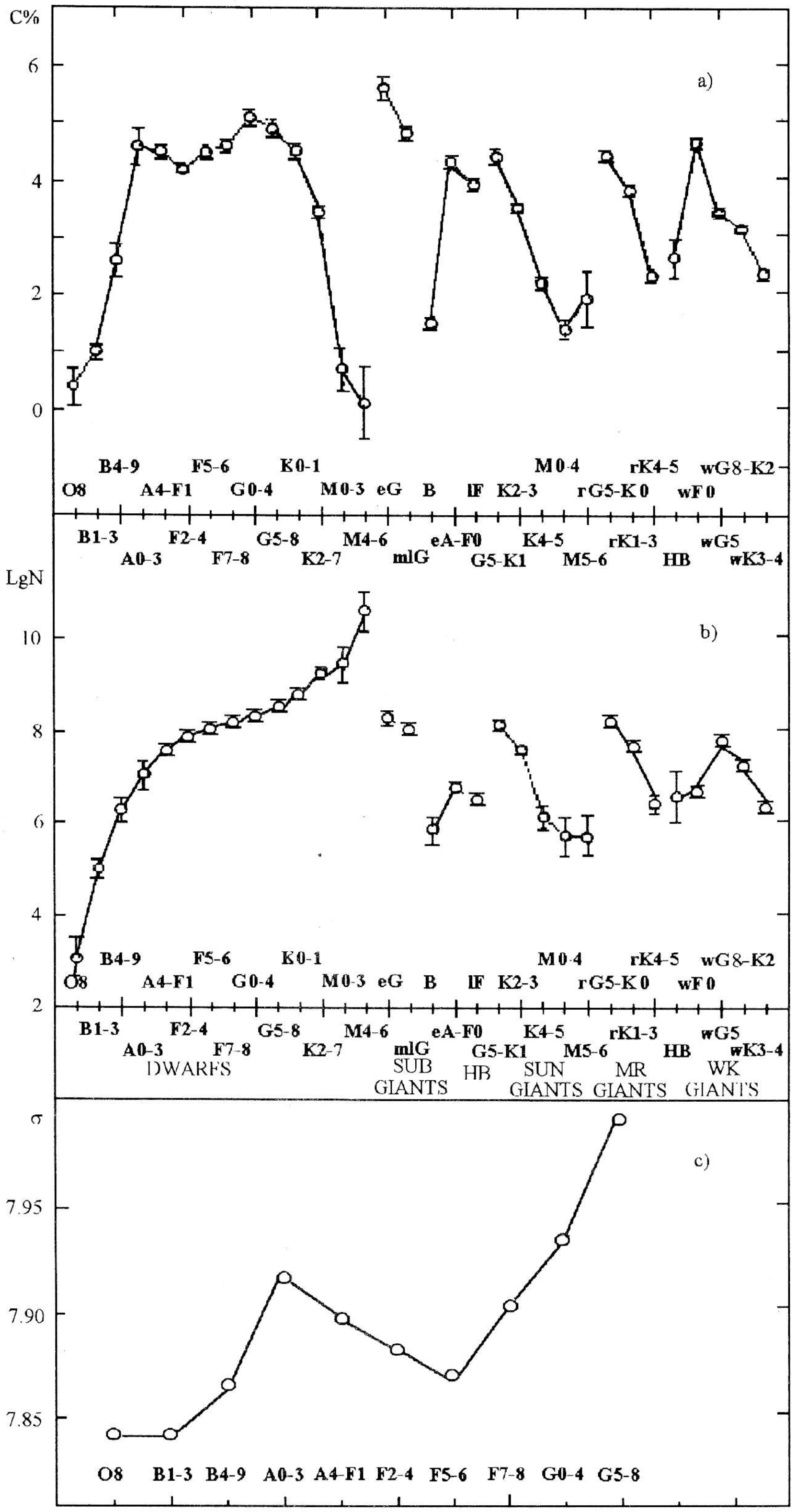


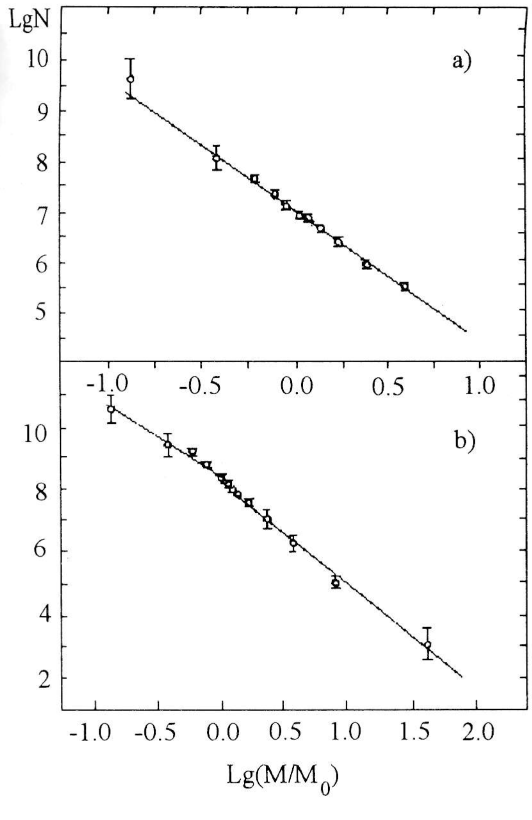
Figure 4: *The model for the galaxy NGC 628. a,b) Designations are the same as in Fig.2. c) Dynamics of th discrepancy from the computed model against the BRS group from which the MS begins.*

Table 1:

Object	R(kpc)	M_v	$\mathcal{M}$ (in $\mathcal{M}_\odot$)	N	$\mathcal{M}/L_v$
NGC 205	640	8.20	$7.24 \cdot 10^8$	$4.45 \cdot 10^9$	2.3 ± 1.0
NGC 628	10000	10.35	$2.26 \cdot 10^{10}$	$1.45 \cdot 10^{11}$	3.5 ± 3.0

Table 2:

Object	IMF		Metallicity		Age(bill.y.)	
	Upper part of MS	Lower part of MS	$[Fe/H]_{mid(l)}$	$[Fe/H]_{mid(m)}$	Young popul.	Old popul.
NGC 205	$x_1 = 2.62$	$x_2 = 2.62$	−0.82	−0.85	0.316	~ 11
	$P_1 = 7.01$	$P_2 = 7.01$	±0.07	±0.07	±0.004	±1
NGC 628	$x_1 = 3.20$	$x_2 = 2.53$	−0.55	−0.162	0.0023	4.85
	$P_1 = 8.25$	$P_2 = 8.44$	±0.04	±0.002	±0.0001	±0.04



igure 5: *The observed mass function (dots) and omputed IMF (straight line) for the galaxies: a) VGC205; b) NGC628.*

ld component the mass weighted mean metallicity alue of the galaxy was used.

6. Conclusion

Thus, the developed version of population synthesis allows one to determine with a sufficient accuracy the most important physical characteristics of an extragalactic object with complex stellar population composition from its integral spectrum. Proceeding from the character of the libraries of stellar spectra we currently have available, the system "SYNTHES" presented here is intended for analysis of normal galaxies (from elliptical to late spirals) in which nonstellar emission sources are missing.

However, the proposed technique of population synthesis enables solution of a wider range of problems with minor modification of the system. The application of weight functions with the aim of picking out for synthesis only individual portions of the galactic spectrum will make it possible to perform modelling for LINER galaxies with separation of their emission spectrum.

In order to extand the scope of applications of this system, it is, first of all, necessary to combine two available banks of observational stellar spectra and then to create an original library of reference spectra which will include, apart from the spectra of stellar populations, a set of spectra of nonstellar component sources. Then one will have an opportunity to solve similar synthesis problems for a wider class of extragalactic objects: active and anomalous galaxies, protogalaxies, dwarf galaxies, etc.

And finally, to improve the accuracy of determination of some major astrophysical parameters (turn–off point, age of a galaxy), it seems promising to combine the techniques of population and evolution synthesis, i.e. to use spectra of star clusters of the same age obtained from evolution synthesis as the BRS elements.

It is a pleasure to thank Yu.N. Mishurov for taking part in developing the technique, valuable discussions and critical remarks. I also express my cincere gratitude to A.J. Pickles for the libraries of stellar spectra made available to me; to V.A. Lipovetsky for support and assistance in realizing the project, to I.S. Balinskaya for helpful discussion of results, and also to the referee for valuable comments.

References

Afanasiev V.L., Lipovetsky V.A., Mihailov V.P., Nazarov E.A., Shapovalova A.I.: 1991, Bull. Spec. Astrophys. Obs., **31**, 128.
Arimoto N., Yoshii Y.: 1987, Astron. Astrophys., **173**, 23.
Bica E.: 1988, Astron. Astrophys., **195**, 76.
Bica E., Alloin D.: 1986, Astron. Astrophys., **162**, 21.
Bica E., Alloin D., Schmidt A.A.: 1990, Astron. Astrophys., **228**, 23.
Buonanno R., Corsi C.E., Fusi Pecci F., Hardy E., Zinn R.: 1985, Astron. Astrophys., **152**, 65.
Carter D., Sadler E.M.: 1990, Mon. Not. R. Astron. Soc., **245**, 12.
Demarque P., Green E.M., King C.:1987, The revised Yale isochrones and luminosity function. New Haven: Yale University observatory.
Faber S.M.: 1972, Astron. Astrophys., **20**, 361.
Forsythe G.E., Malcolm M.A., Moler C.B.: 1980, Computer methods for mathematical computations (in Russian). Moscow: Nauka, 91.
Gunn J.E., Stryker L., Tinsley B.M.: 1981, Astrophys. J., **249**, 48.
Jacoby G.H., Hunter D., Carol C.: 1984, Astrophys. J. Suppl. Ser., **56**, 257.
Lowson C.L., Hanson R.J.: 1986, Solving least squares problems (in Russian), Moscow, Nauka, 91.
O'Connell R.W.: 1976, Astrophys. J., **206**, 370.
O'Connell R.W.: 1980, Astrophys. J., **236**, 430.
Pickles A.J.: 1985, Astrophys. J., **296**, 340.
Pickles A.J.: 1985, Astrophys. J. Suppl. Ser., **59**, 33.
Schild R.E.: 1977, Astron. J., **82**, no.5, 337.
Shchigolev V.M.: 1969, Mathematical processing of observations (in Russian), Moscow, Nauka.
Somov N.N.: 1986, Astrofiz.Issled. (Izv.SAO), **22**, 73.
Spinrad H., Taylor B.J.: 1971, Astrophys. J. Suppl. Ser., **22**, 445.
Tinsley B.M.: 1980, Fundamentals Cosmic Phys., **5**, 287.
Williams T.W.: 1976, Astrophys. J., **209**, 716.